

High-fidelity quantum driving

Mark G. Bason¹, Matthieu Viteau¹, Nicola Malossi², Paul Huillery^{1,3}, Ennio Arimondo^{1,2,4},
Donatella Ciampini^{1,2,4}, Rosario Fazio⁵, Vittorio Giovannetti⁵, Riccardo Mannella⁴
and Oliver Morsch^{1*}

Accurately controlling a quantum system is a fundamental requirement in quantum information processing and the coherent manipulation of molecular systems. The ultimate goal in quantum control is to prepare a desired state with the highest fidelity allowed by the available resources and the experimental constraints. Here we experimentally implement two optimal high-fidelity control protocols using a two-level quantum system comprising Bose–Einstein condensates in optical lattices. The first is a short-cut protocol that reaches the maximum quantum-transformation speed compatible with the Heisenberg uncertainty principle. In the opposite limit, we realize the recently proposed transitionless superadiabatic protocols in which the system follows the instantaneous adiabatic ground state nearly perfectly. We demonstrate that superadiabatic protocols are extremely robust against control parameter variations, making them useful for practical applications.

The need to control the dynamics of a quantum system is common to many different areas of science¹, ranging from the coherent manipulation of molecular systems² to high-precision measurements³ and quantum computation⁴. Quantum control may aim at reaching a given target state, as in the cooling of atomic ensembles⁵ and nanomechanical oscillators⁶, or at tracking the instantaneous ground state of a system during its evolution, as in adiabatic quantum computation⁷. For each of these tasks an optimum strategy can be designed. In most applications the requirements on the fidelity of the final state is stringent, and in principle one aims at perfect fidelity.

Here we investigate high-fidelity quantum control protocols for the ‘simplest non-simple quantum problem’⁸, that is, the evolution of a two-level system in a time T , as illustrated in Fig. 1a (energies and times are expressed in the natural units of our physical system, see Methods). Here two states $|0\rangle$ and $|1\rangle$, the diabatic levels, are coupled via a time-dependent Landau–Zener (LZ) Hamiltonian of the form^{9,10}

$$\mathcal{H}_{LZ} = \Gamma(\tau)\sigma_z + \omega(\tau)\sigma_x \quad (1)$$

($\sigma_{z,x}$ being the Pauli operators with $\sigma_x|0\rangle = |1\rangle$) characterized by the instantaneous adiabatic levels of the system $|\psi_{g,e}(\tau)\rangle$, where ω is the coupling between the diabatic levels. Figure 1 shows the energy spectrum of the system for a constant ω and a linear dependence of $\Gamma(\tau)$ on the rescaled time $\tau = t/T \in [0, 1]$ with $\Gamma(1) = -\Gamma(0)$, which features an avoided crossing of the adiabatic levels $|\psi_{g,e}(\tau)\rangle$ at $\Gamma(\tau) = 0$ with an energy gap of 2ω .

Our goal is to design a control protocol that drives the system through the anticrossing point in such a way that at the end of the evolution ($\tau = 1$) the final state $|\Psi_{\text{fin}}\rangle$ is as close as possible to the adiabatic ground state $|\psi_g(1)\rangle$, aiming for a final fidelity $\mathcal{F}_{\text{fin}} = |\langle \Psi_{\text{fin}} | \psi_g(1) \rangle|^2 = 1$. Assuming that the system is initially prepared in the adiabatic ground state $|\Psi_{\text{ini}}\rangle = |\psi_g(0)\rangle$, there are infinitely many paths in Hilbert space connecting $|\psi_g(0)\rangle$ with

$|\psi_g(1)\rangle$. In this work we concentrate on two special classes of such paths. Specifically, we first consider those that minimize the time T of the transformation for constant ω and hence reach the ultimate speed limit imposed by quantum mechanics^{11–15}. Subsequently, allowing for the possibility of a time-varying ω , we analyse those protocols that ensure a perfect following of the adiabatic ground state $|\psi_g(\tau)\rangle$ for all τ (refs 16–18). We will call the latter ‘superadiabatic’ with reference to Berry’s work on transition histories and the superadiabatic basis¹⁹.

Even though the results we present here are general and in principle apply to any two-level quantum system, in our analysis we will focus on an experimental implementation which permits a practical test of the various protocols. We realize an effective two-level system using Bose–Einstein condensates (BECs) in an accelerated optical lattice²⁰ (Fig. 1b), which allows for fast and stable control of the system parameters. Under appropriate conditions the wavefunction of the BEC in the periodic potential of the optical lattice can be approximated by considering only the two lowest energy bands^{21,22}, which exhibit an avoided crossing at the edge of the first Brillouin zone (Fig. 1c) and thus realize the Hamiltonian of equation (1), where $\Gamma(\tau)$ and $\omega(\tau)$ can be controlled through the quasimomentum q and the depth V_0 of the optical lattice, respectively (for details see Methods). The system is initially prepared in the lowest energy band of the lattice with $q = 0$ (corresponding to $|\Psi_{\text{ini}}\rangle$), and the target is to reach $|\Psi_{\text{fin}}\rangle$ after an evolution of duration T .

We begin by considering a protocol with constant ω which, in principle, drives the system from $|\Psi_{\text{ini}}\rangle$ to $|\Psi_{\text{fin}}\rangle$ with fidelity $\mathcal{F}_{\text{fin}} = 1$ in the shortest possible time T_{min} . By analogy with the equivalent classical case this kind of protocol has been called the ‘quantum brachistochrone’²³. Imposing only the constraint that ω be constant (otherwise $T_{\text{min}} \rightarrow 0$ as $\omega \rightarrow \infty$), we find that the protocol shown in Fig. 2c minimizes T . This ‘composite pulse’ protocol (Methods), in close analogy to composite pulses in nuclear magnetic resonance (ref. 24), represents half a Rabi oscillation with frequency ω at $\tau = 0.5$, preceded and followed by two short pulses (in theory

¹INO-CNR, Largo Pontecorvo 3, 56127 Pisa, Italy, ²CNISM UdR, Dipartimento di Fisica ‘E. Fermi’, Università di Pisa, Largo Pontecorvo 3, 56127 Pisa, Italy, ³Laboratoire Aimé Cotton, Univ. Paris-Sud 11, Campus d’Orsay Bat. 505, 91405 Orsay, France, ⁴Dipartimento di Fisica ‘E. Fermi’, Università di Pisa, Largo Pontecorvo 3, 56127 Pisa, Italy, ⁵NEST, Scuola Normale Superiore, and Istituto di Nanoscienze—CNR, 56126 Pisa, Italy. *e-mail: morsch@df.unipi.it.

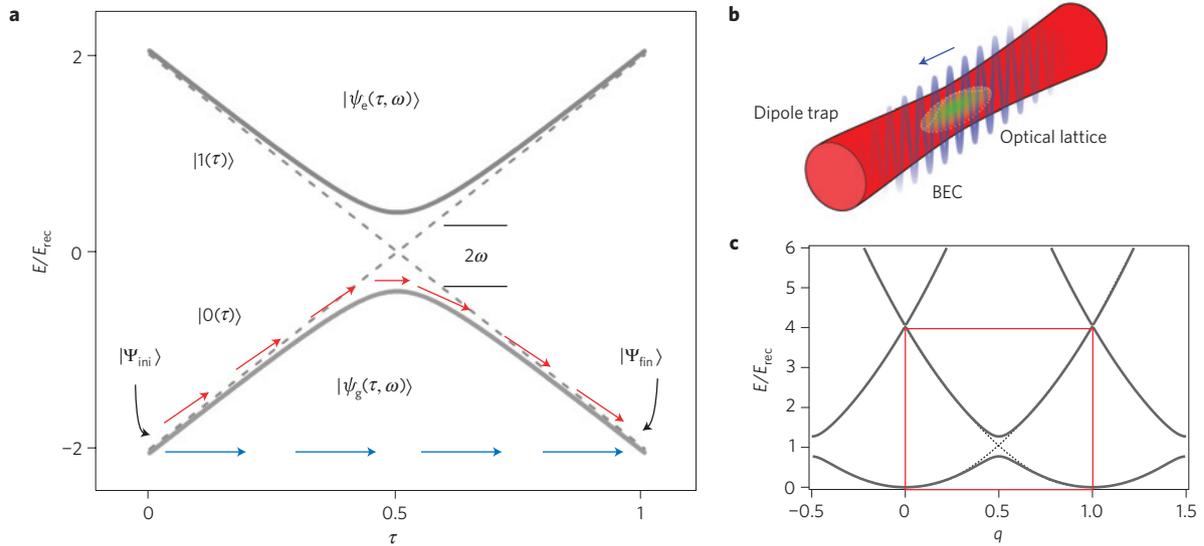


Figure 1 | Schematic of a two-level quantum system and experimental realization. a, Two-level system with crossing energy levels for a linear dependence $\Gamma(\tau) = 4E_{\text{rec}}(\tau - 1/2)$ (where E_{rec} is the natural energy scale of the system, for example, the recoil energy in the case of condensates in optical lattices). The bare (diabatic) states $|0\rangle$ and $|1\rangle$ are coupled to give the adiabatic states $|\psi_g\rangle$ and $|\psi_e\rangle$. The adiabatic states have an energy gap of 2ω where the diabatic levels cross (at $\tau = 0.5$, $E = 0$). The arrows indicate the two extreme protocols discussed in this paper: the ‘short cut’ composite pulse protocol reaching the quantum speed limit (blue line) and the superadiabatic protocol (red line) for which the system perfectly follows the instantaneous ground state. **b**, Experimental realization using a BEC. The condensate is held in an optical dipole trap and loaded into an optical lattice that can be accelerated to vary the quasimomentum of the system. **c**, Band structure of the condensate in the optical lattice. The red box indicates the range of quasimomenta used to realize an effective two-level system in the two lowest energy bands.

delta functions) with a pulse area of $\pi/4$ (see also ref. 25). The experimentally measured minimum times (Fig. 2d) for reaching the target state approach the quantum speed limit time T_{qsl} , also known as the Fleming or Bhattacharyya bound¹⁴, given by

$$T_{\text{qsl}} = \frac{\arccos|\langle \Psi_{\text{fin}} | \Psi_{\text{ini}} \rangle|}{\omega} \quad (2)$$

One can assess the performance of the composite pulse protocol by comparing it to the paradigmatic LZ protocol and to the locally adiabatic protocol proposed by Roland and Cerf (RC) in the context of adiabatic quantum computation²⁶. In the LZ problem with constant ω and linearly varying $\Gamma(\tau)$ the system initially prepared in the adiabatic ground state undergoes tunnelling to the excited state with a finite probability, leading to a fidelity which, even in the absence of experimental imperfections, is bounded by $\mathcal{F}_{\text{fin}} = 1 - \exp(-\pi T \omega^2/4) < 1$. Stricter constraints are imposed in the RC approach. As well as the condition that ω be constant they demand local adiabatic following to within some small deviation ϵ , that is, during the entire protocol $\mathcal{F}(\tau) \geq 1 - \epsilon^2$ (Methods). In both the LZ and the RC protocols the time to achieve perfect adiabaticity, that is, $\mathcal{F}_{\text{fin}} = 1$, diverges and hence we measure the time to reach $\mathcal{F}_{\text{fin}} = 0.9$ as a function of ω instead (shown as a dotted line in Fig. 2e). The results of the comparison are shown in Fig. 2d. It is evident that whereas the LZ protocol is more than an order of magnitude slower than the composite pulse protocol for small ω , the RC protocol reaches $\mathcal{F}_{\text{fin}} = 0.9$ in a time that is only 10–50% above the lower limit given by the bound (2).

At the opposite extreme of the quantum control spectrum, rather than minimizing the total time one can maximize the adiabaticity during the protocol. The composite pulse protocol described above was obtained through optimization given a constraint on ω , but it is also possible to analytically calculate protocols that ensure $\mathcal{F}(\tau) = 1$ during the entire evolution. The reasoning behind such a transitionless superadiabatic (or counterdiabatic^{16,17}) protocol is that for a given time-varying Hamiltonian \mathcal{H} it is always

possible to construct an auxiliary Hamiltonian \mathcal{H}_s that cancels the non-adiabatic part of the evolution under \mathcal{H} alone. It thus ensures transitionless adiabatic following such that the system evolving under $\mathcal{H} + \mathcal{H}_s$ always remains in the instantaneous adiabatic ground state of \mathcal{H} with 100% probability^{16,19}, even for a finite duration of the protocol. In general \mathcal{H}_s can be written as

$$\mathcal{H}_s(t) = i\hbar \sum_n (|\partial_t n(t)\rangle \langle n(t)| - |n(t)\rangle \langle \partial_t n(t)|) / 2$$

where $|n(t)\rangle$ are the eigenstates of the original Hamiltonian \mathcal{H} . For a two-level system of the form (1) one finds that

$$\mathcal{H}_s(t) = \frac{\hbar}{2} \frac{\partial \phi}{\partial t} \sigma_y$$

where $\phi = \arctan(\omega(t)/\Gamma(t))$.

This means that to make a given evolution of a two-level system perfectly adiabatic one needs to add an interaction term corresponding to a σ_y Pauli matrix²⁷. In practice, \mathcal{H}_s can be implemented by introducing an additional interaction into the system, for example, through an extra laser or microwave field. In the case of atoms in an optical lattice considered in this work, \mathcal{H}_s can be realized by adding a second optical lattice shifted with respect to the first by $d_L/4$ (d_L is the lattice spacing). It can be shown, however, that the effect of this extra field can also be achieved through an appropriate transformation $\Gamma \rightarrow \Gamma'$ and $\omega \rightarrow \omega'$ (Methods), such that no extra field is necessary. This transformation, which should be independent of the physical system under consideration, means that the resulting protocol is intrinsically more stable, as there will be no problems associated, for example, with phase fluctuations between the fields^{16,17}.

For the standard LZ protocol the general shape of the required transformation is shown in Fig. 3a (for details see Methods), together with the result of an experiment in which for different sweep durations T the fidelity \mathcal{F}_{fin} was measured both for

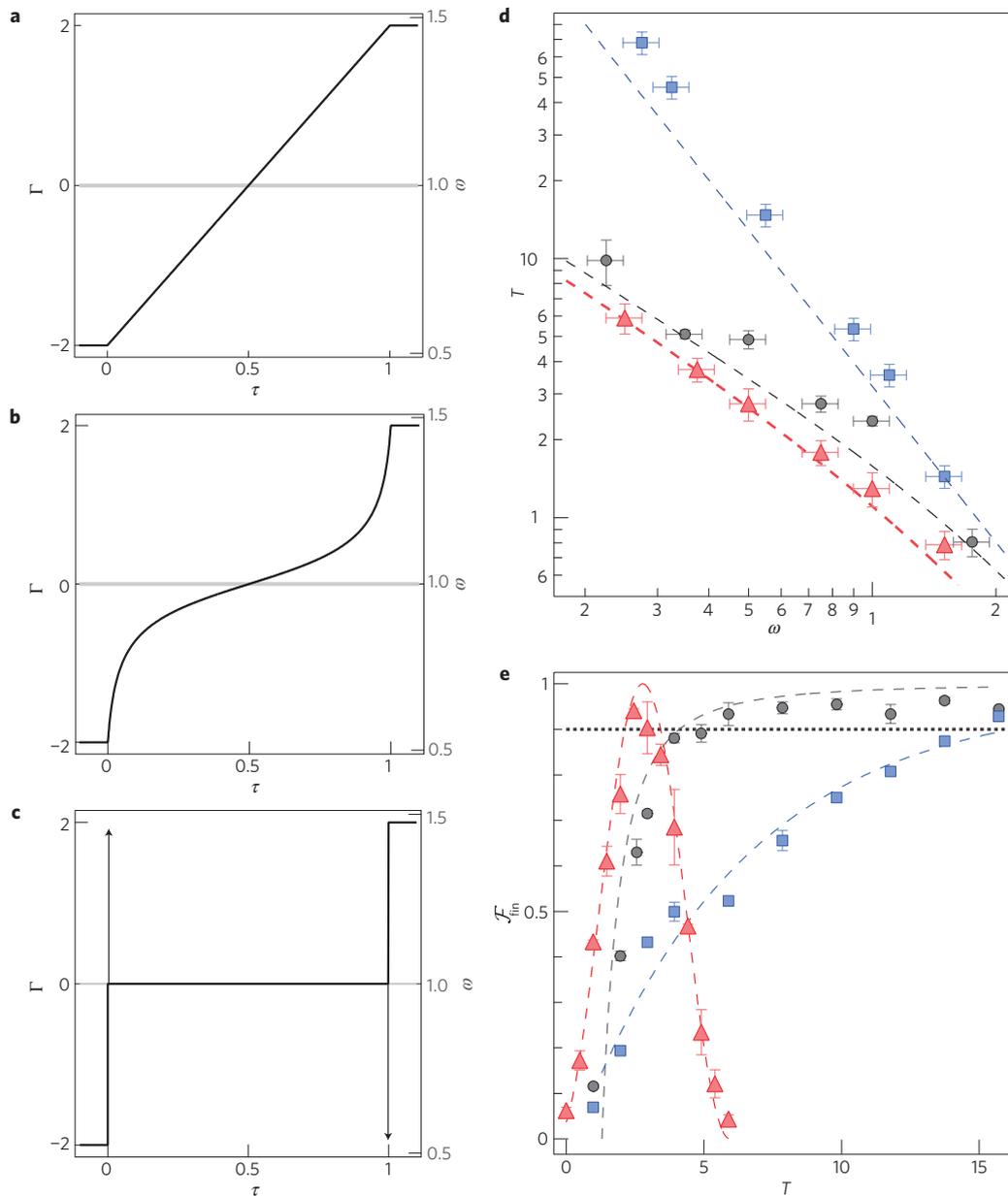


Figure 2 | Comparison between the driving protocols. a–c, The time-dependence of L and ω is shown for the LZ (a), RC (b) and composite pulse (c) protocols. **d,** The time needed to achieve optimal fidelity for the composite pulse protocol (red triangles) and the minimum time to achieve $\mathcal{F}_{\text{fin}} = 0.9$ in the RC protocol (grey circles) and the LZ protocol (blue squares). The horizontal error bars reflect the estimated calibration uncertainty of the lattice depth V_0 and, hence, of ω . **e,** Fidelity of the final state as a function of the duration for the composite pulse protocol (red triangles), the RC protocol (grey circles) and the LZ protocol (blue squares). In **d** and **e**, the dashed lines are theoretical predictions. The horizontal dotted line in **e** represents the 0.9 fidelity threshold. Note that in the composite pulse protocol $\mathcal{F}_{\text{fin}} = 1$ is not always reached because of an interaction-induced loss of coherence in the BEC occurring predominantly close to the avoided crossing, where the system spends most of its time in the composite pulse protocol. T was extracted instead from the position of the maximum in \mathcal{F}_{fin} , typically 0.95 ± 0.03 . All experimental data in **e** are for $\omega = 0.5$. Vertical error bars are one s.d.

the linear LZ protocol and for its superadiabatic counterpart (Fig. 3c). We find that in the superadiabatic case $\mathcal{F}_{\text{fin}} = 0.98 \pm 0.01$ for all T . Furthermore, a time-resolved measurement of $\mathcal{F}(\tau)$ during the sweep (Fig. 3d) shows that, to within the experimental error, the system stays in the ground adiabatic state at all times with $\mathcal{F}(\tau) = 0.98 \pm 0.02$ (see Methods for a discussion of experimental errors).

In principle, the superadiabatic transformation of L and ω can be calculated for any sweep protocol. There are, however, special cases that are of particular interest. For example, a protocol for which the correction in L vanishes, that is, for which $L' = L$ (except at the beginning and the end of the sweep), can be expected

to be more robust to variations in its parameters. This is the case for the superadiabatic ‘tangent protocol’ (Methods) shown schematically in Fig. 3b. As in the case of the linear LZ protocol with superadiabatic corrections, in the tangent protocol we measure $\mathcal{F}(\tau) = 0.99 \pm 0.02$ throughout the entire protocol. This figure was confirmed by repeating the protocol four times, which gave an overall fidelity $\mathcal{F}_{\text{fin}}^{\text{tot}} \approx 0.94$, in agreement with $\mathcal{F}_{\text{fin}} \approx 0.99$ for a single protocol. As we estimate the non-adiabaticity of the preparation and measurement protocols to be on the order of 1% (see Methods), this result is compatible with an actual infidelity $1 - \mathcal{F}_{\text{fin}}$ of the superadiabatic protocol at the 10^{-3} level found in our numerical simulations when taking into account the non-ideal experimental

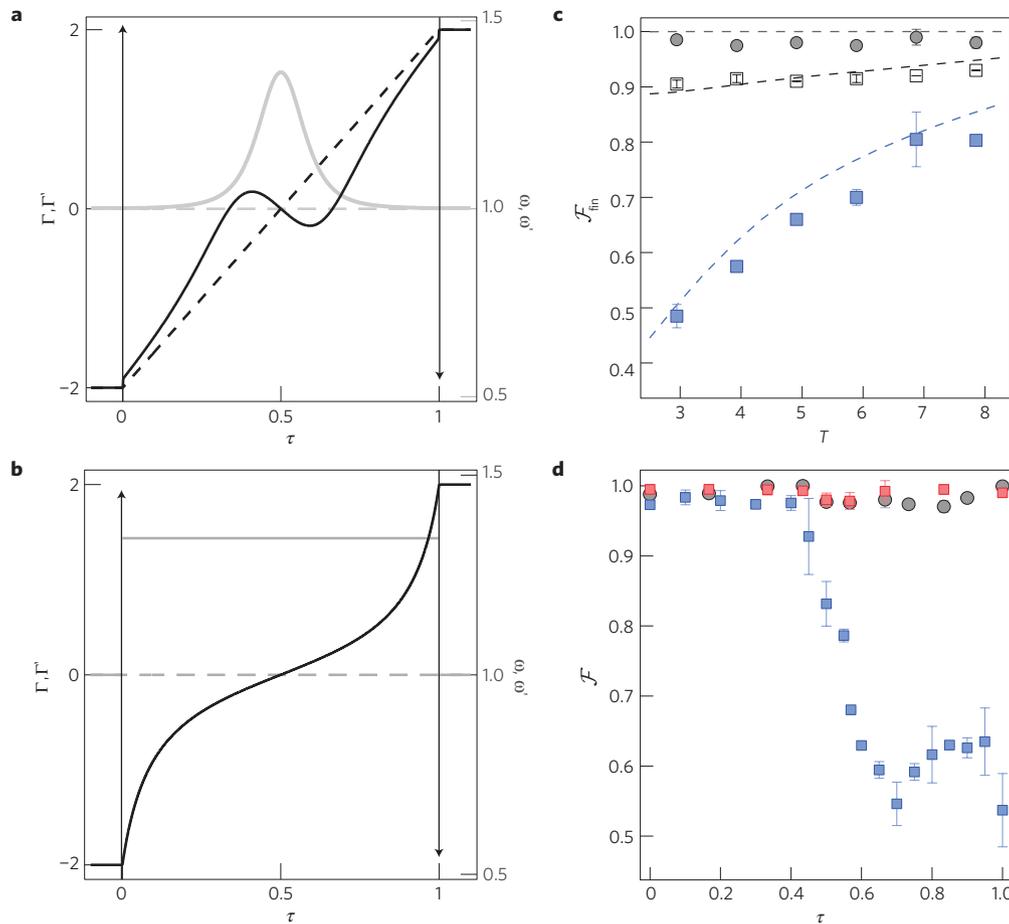


Figure 3 | Superadiabatic dynamics in a two-level system. **a,b**, Original (dashed lines) and superadiabatic (solid lines) protocols for the linear (**a**) and tangent (**b**) cases. **c**, Fidelity as a function of the duration of the protocol for the LZ protocol (blue squares) and the superadiabatic linear protocol (grey circles). For comparison, the results of a linear LZ sweep in which only ω is transformed (equation (5)) is also shown (open squares). The dashed lines are theoretical predictions. **d**, Fidelity during the protocol for the superadiabatic linear (grey circles), superadiabatic tangent (red squares) and LZ (blue squares) protocols. For the superadiabatic protocols $\mathcal{F}(\tau) = 0.98 \pm 0.02$ during the entire protocol. All experimental data are for $\omega = 0.55$. Error bars are one s.d.

pulses at the beginning and the end of the protocols. This level of fidelity is comparable to the record fidelities of $\mathcal{F}_{\text{fin}} = 0.994$ reported for two-qubit gate transformations in trapped ions (see, for example ref. 28, and references therein) and to $\mathcal{F}_{\text{fin}} \approx 0.99$ achieved in recent experiments on state preparation in trapped ions using rapid adiabatic passage²⁹. It would be interesting, therefore, to apply the methods demonstrated in a proof-of-principle fashion in this paper to other systems and compare the results to, for example, stimulated Raman adiabatic passage techniques³⁰.

To test the sensitivity of the superadiabatic tangent protocol to a (simulated) variation in the control parameters, we varied both ω and T around their optimum values and measured \mathcal{F}_{fin} . The results are summarized in Fig. 4a, which shows clearly that the superadiabatic tangent protocol is extremely robust with respect to an increase in T or ω , with $\mathcal{F}_{\text{fin}} \gtrsim 0.99$ for increases in the values of the experimental parameters up to 100% ($1 - \mathcal{F}_{\text{fin}} < 10^{-3}$ in our simulations). For large reductions in T or ω the fidelity naturally drops sharply, as otherwise the quantum speed limit would be violated. This suggests a practical strategy for increasing stability by choosing, for example, a value for ω that is 30–40% larger than the ideal value to achieve stability for both positive and negative $\Delta\omega$.

Finally, we compare the speed of the superadiabatic tangent protocol with the composite pulse protocol as a function of ω' (for the composite pulse protocol $\omega' = \omega$). Solving equation (6) for T gives a total time for the superadiabatic tangent protocol that

depends on both ω and ω' (with $\omega' > \omega$). It is, therefore, possible to minimize T for a given ω' by choosing an appropriate value for ω . The result of this minimization is shown in Fig. 4b. Surprisingly, the minimum value of T as a function of ω' for the superadiabatic tangent protocol lies below the LZ and RC times and is quite close to the quantum speed limit, meaning that the ‘penalty’ in terms of speed for the requirement of perfect adiabatic following is less than one might at first expect. In fact, for small values of ω' one can formally let $\omega \rightarrow 0$ in the expression for ω' (see equation (6) in Methods), giving $T = \pi/2\omega'$, which coincides with the quantum speed limit for orthogonal states with $|\langle \psi_g(1) | \psi_g(0) \rangle| = 0$.

In summary, we have explored high-fidelity quantum control protocols for the evolution of a two-level quantum system, ranging from the speed-limited to the superadiabatic regime. The superadiabatic transformations make it possible to readily implement protocols ensuring near-perfect adiabatic following in a variety of existing applications. In practice, of course, the choice of protocol will depend on the boundary conditions and physical limitations of the system under consideration. If both T and ω can be controlled (to within some limits), the superadiabatic protocols provide the possibility of state preparation with close to 100% fidelity, with high stability against parameter variations. That stability, in particular, should prove useful for improving existing adiabatic control protocols that already achieve high fidelities²⁹.

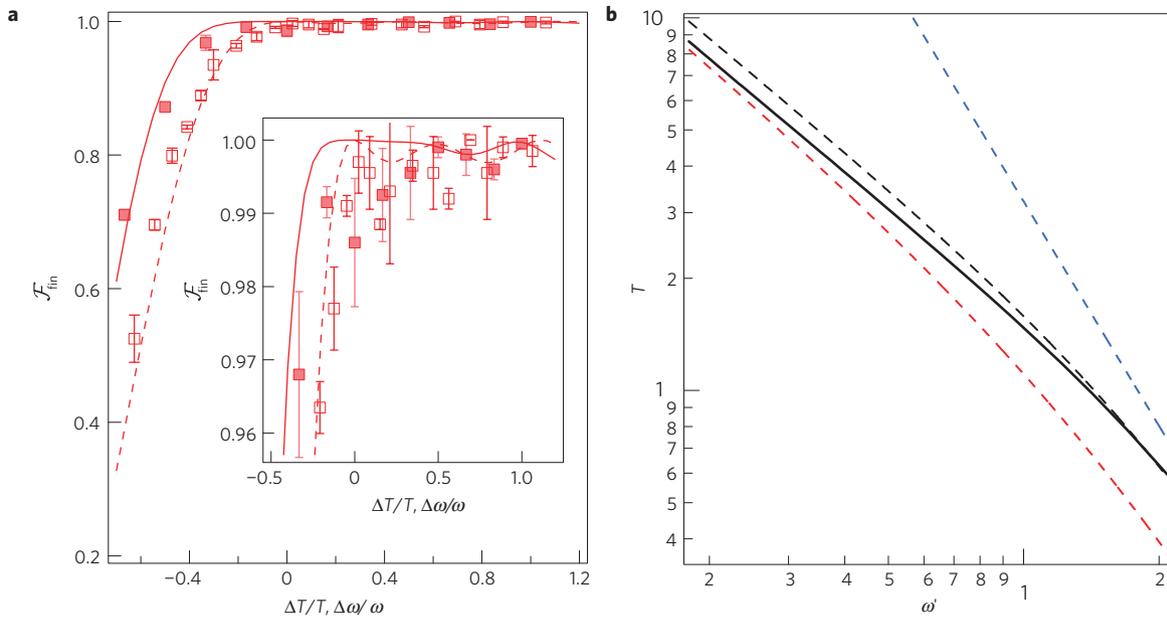


Figure 4 | Robustness and speed of the superadiabatic tangent protocol. **a**, Fidelity of the superadiabatic tangent protocol as a function of the relative deviation of T (filled red squares) and ω (open red squares) from their ideal values: $\omega = 0.5$, $T = 5.9$. The solid and dashed lines are the respective numerical simulations. Inset: Detail of the graph, highlighting the stability of the protocol for $\Delta T/T, \Delta\omega/\omega > 0$. Error bars are one s.d. **b**, Comparison of the duration of superadiabatic tangent protocols with the composite pulse (dashed red line), RC (dashed black line) and LZ (dashed blue line) protocols. The solid black line shows the minimum time for the superadiabatic tangent protocol as a function of ω' .

Methods

Bose–Einstein condensates in optical lattices. Bose–Einstein condensates are loaded into optical lattice potentials²⁰ of the form $(V_0/2)\cos(2\pi x/d_L + \phi(t))$. In the limit of small lattice depths $V_0 \lesssim 5 E_{\text{rec}}$ (here the recoil energy $E_{\text{rec}} = \hbar\omega_{\text{rec}} = \pi^2 \hbar^2 / 2Md_L^2$ for atoms of mass M , and $\omega_{\text{rec}} = 2\pi \times 3.15$ kHz defines the natural units of energy $\hbar\omega_{\text{rec}}$ and time $1/\omega_{\text{rec}}$ for our system with rubidium atoms), the lowest energy levels are given by the quadratic dispersion relations of free particles with momenta differing by $2\hbar k_L$, coupled through $\omega = V_0/4$. Subtracting the quadratic term in the expression for the energy bands²² leads to the effective Hamiltonian (1), where the time dependence of Γ is achieved through a variation of the quasimomentum q . Experimentally, q and hence $\Gamma = 4\hbar\omega_{\text{rec}}(q - 1/2)$ (in this Methods section we shall use the explicit physical units wherever appropriate) is controlled through the term $\phi(t)$ (by acousto-optic modulators), which can be used to accelerate the lattice. The time dependence of ω is controlled through the power of the lattice laser beams, which determines V_0 . The experimental protocols are carried out using techniques previously developed by us^{21,22}. Time-resolved measurements of the superadiabatic protocols are performed by applying appropriate jumps of the lattice position, the quasimomentum and/or the lattice depth, before measuring in the adiabatic basis (now of the original Hamiltonian \mathcal{H}). The measurements of \mathcal{F}_{fin} reported in this paper carry a statistical error of around ± 1 –2% due to noise in the absorption images used to deduce the number of atoms in the adiabatic levels. A systematic error of $\approx -1\%$ stems from imperfect preparation and (adiabatic) detection of the system, and measurements of $\mathcal{F}(\tau)$ may carry a further systematic error of $\pm 1\%$ as the detection protocols were optimized only for $\tau = 1$.

Driving protocols. For the composite pulse protocol the optimality of the time (2) in the LZ setting was derived in ref. 11 by starting from a guess function similar to the RC protocol (see below), and by running a numerical search aimed at determining the minimal value of T (in that optimization the adiabatic requirement was not enforced). The optimal pulse $\Gamma(\tau)$ associated with such a minimal time corresponds to highly irregular functions which are strongly peaked at the beginning and the end of the protocol while remaining flat and equal to zero for intermediate times. Assuming that no constraints are imposed on $\Gamma(\tau)$ and on its first derivative one can extrapolate the exact (asymptotic) analytical form of such pulses as

$$\Gamma(\tau) = \begin{cases} -\Gamma_0 & \text{for } \tau = 0 \\ +\Gamma_M & \text{for } \tau \in [0, \tau_0] \\ 0 & \text{for } \tau \in [\tau_0, 1 - \tau_0] \\ -\Gamma_M & \text{for } \tau \in [1 - \tau_0, 1] \\ +\Gamma_0 & \text{for } \tau = 1 \end{cases} \quad (3)$$

where $\Gamma_0 = 2$ as usual, with Γ_M and τ_0 being, respectively, asymptotically large and small quantities which satisfy the condition

$$\Gamma_M \tau_0 = \pi/4 \quad (4)$$

For $\Gamma_M \gg 1$ the evolution described by the pulse (3) corresponds to first applying a fast (instantaneous) clockwise σ_z pulse around the z -axis to the system, which takes the point on the Bloch sphere lying in the x - z plane and, in the short time τ_0 , rotates it into the y - z plane. The system then undergoes a rotation at frequency ω around the x axis for a time $1 - 2\tau_0$, then finally another instantaneous σ_z pulse rotates the x - z plane back to its initial position.

The exact transfer of $|\Psi_{\text{ini}}\rangle$ to $|\Psi_{\text{fin}}\rangle$ is achieved by choosing T such that

$$T(1 - 2\tau_0) = \frac{\arccos(\langle \Psi_{\text{fin}} | \Psi_{\text{ini}} \rangle)}{\omega}$$

which coincides with the r.h.s. of equation (2) by letting $\tau_0 \rightarrow 0$ while keeping (4). We note here that although the fact that the composite pulse protocol realizes the quantum speed limit suggests that this time is optimal, a formal proof for this is still lacking.

For the RC protocol we adapt the analysis of Roland and Cerf²⁶ to the case of the Hamiltonian \mathcal{H} of equation (1) under the assumption that the coupling ω is kept constant. In this protocol the requirement is to keep the evolution adiabatic in each infinitesimal time interval. At any instant of the evolution the fidelity of the state with the instantaneous ground state is required to be $\mathcal{F}(\tau) = |\langle \psi(\tau) | \psi_g(\tau) \rangle|^2 = 1 - \epsilon^2$, from which it follows that the total time of the protocol is

$$T(\epsilon) = \frac{1}{\epsilon\omega} \frac{1}{\sqrt{4 + \omega^2}}$$

for a time-dependence of $\Gamma(t)$ of the form

$$\Gamma(\tau) = \frac{4\epsilon\omega^2 T(\epsilon)(\tau - 1/2)}{\sqrt{1 - 16\epsilon^2\omega^2 T(\epsilon)^2(\tau - 1/2)^2}}$$

(as shown in Fig. 2b).

The superadiabatic protocols are obtained by recasting the Hamiltonian $\mathcal{H}(\tau) + \mathcal{H}_L(\tau) = \Gamma(\tau)\sigma_z + \omega(\tau)\sigma_x + (\hbar/2)(\partial\phi/\partial t)\sigma_y$ in the form $\mathcal{H}' = \Gamma'(\tau)\sigma_z + \omega'(\tau)\sigma_x$, eliminating the need for an extra potential that realizes the σ_y -term. For the system studied in this paper the necessary transformations $\Gamma \rightarrow \Gamma'$ and $\omega \rightarrow \omega'$ can be derived by observing that the Pauli matrices σ_x and σ_y are simply related by the spatial displacement operator $\hat{U}_d(\delta x)$ with $\delta x = d_L/4$, where d_L is the lattice constant. In the basis of the plane waves $\exp(\pm ikx)$ defining

the two-state subspace $\{|0\rangle, |1\rangle\}$, this operator can be written as

$$\hat{U}_d(d_L/4) = \begin{pmatrix} e^{\frac{\pi}{4}i} & 0 \\ 0 & e^{-\frac{\pi}{4}i} \end{pmatrix}$$

and hence

$$\sigma_y = \hat{U}_d^\dagger(d_L/4)\sigma_x\hat{U}_d(d_L/4)$$

This means that the σ_y -term in the transitionless driving protocol can be realized by adding a second periodic potential shifted by $d_L/4$ and accelerated in the same way as the first one. As the sum of two periodic potentials of the same periodicity is again a periodic potential with a modified phase and amplitude, we can write the combined potential

$$V_{\text{tot}}(x) = V_0 \cos^2\left(\frac{\pi x}{d_L}\right) + 4\alpha \cos^2\left(\frac{\pi(x - \frac{d_L}{4})}{d_L}\right)$$

as

$$V_{\text{tot}}(x) = \text{const.} + 2\sqrt{\left(\frac{V_0}{2}\right)^2 + (2\alpha)^2} \cos^2\left(\frac{\pi(x - \frac{\beta d_L}{2\pi})}{d_L}\right)$$

where $\alpha = (\hbar/2)(\partial\phi/\partial t)$ and $\beta = \arctan(4\alpha/V_0)$. From this equation, the transformation rule for $\omega = V_0/4$ can be read off immediately, while the time derivative of the displacement term $(\beta d_L/2\pi)$ gives the correction to the quasimomentum

$$q'(t) = q(t) - \frac{\dot{\beta}}{8\omega_{\text{rec}}}$$

from which the transformation rule for Γ is calculated using the relation $\Gamma = 4\hbar\omega_{\text{rec}}(q - 1/2)$. The complete transformation then reads (in rescaled units)

$$\Gamma' = \Gamma - \frac{1}{2} \frac{d}{dt} \left[\arctan\left(\frac{\Gamma\dot{\omega} - \omega\dot{\Gamma}}{2\omega(\Gamma^2 + \omega^2)}\right) \right]$$

$$\omega' = \omega \sqrt{1 + \frac{(\Gamma\dot{\omega} - \omega\dot{\Gamma})^2}{4\omega^2(\Gamma^2 + \omega^2)^2}} \quad (5)$$

where the discontinuities in the expression in square brackets in Γ' at the beginning and the end of the protocol give rise to delta functions, which can be realized in practice using large but finite corrections $\Delta\Gamma_M$ for a short duration Δt such that

$$\Delta t \Delta\Gamma_M = \mp \frac{1}{2} \arctan\left(\frac{\Gamma\dot{\omega} - \omega\dot{\Gamma}}{2\omega(\Gamma^2 + \omega^2)}\right)$$

where the $-$ and $+$ signs refer to the corrections at the beginning and the end of the protocol, respectively.

The two superadiabatic protocols considered in this paper are the superadiabatic linear protocol with

$$\Gamma'(\tau) = \Gamma(\tau) - \frac{4(\tau - \frac{1}{2})}{T^2[(\tau - \frac{1}{2})^2 + \frac{1}{2}\omega^2] + 1}$$

$$\omega'(\tau) = \omega \sqrt{1 + \frac{1}{T^2(8(\tau - \frac{1}{2})^2 + \frac{1}{2}\omega^2)^2}}$$

and the superadiabatic tangent protocol for which

$$\Gamma'(\tau) = \Gamma(\tau) = \omega \tan\left(2\left(\tau - \frac{1}{2}\right) \arctan\left(\frac{2}{\omega}\right)\right)$$

(apart from the delta functions at the beginning and the end of the protocol) and

$$\omega' = \omega \sqrt{1 + \frac{\arctan(\frac{2}{\omega})^2}{(T\omega)^2}} \quad (6)$$

The latter protocol is found by demanding that $\Gamma'(\tau) = \Gamma(\tau)$ and solving the resulting differential equation.

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Author contributions

M.G.B., M.V., N.M., P.H. and D.C. carried out the experiments; V.G. and R.F. developed the composite pulse protocol; O.M. and R.M. developed the superadiabatic protocols and performed the numerical simulations; E.A., R.F. and O.M. wrote the paper. All authors discussed the results and commented on the manuscript.

Additional information

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